# Apterous Rating System

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In all of the below, Roman letters ($a, b, c \ldots$) are used to represent variables that depend on the particular game in question. Greek letters ($\alpha, \beta, \gamma \ldots$) are constants that are tweaked to balance the significance of various factors within the rating system itself, and are the same for all games rated within a particular system. The values that Apterous actually uses are given at the end.

## 1 Provisionally rated players

A player is said to have a *provisional rating* for their first $\omega$ games, and a *full rating* thereafter. (Apterous currently takes $\omega = 11$.) While a player is provisional, their rating is simply taken to be

$$r = a + \tau \ln \frac{w}{l}$$

where $w$ and $l$ are the number of wins and losses this player has accumulated, and $a$ is the average rating of the players they have faced so far. Of course this rating can vary hugely with just small changes in $w$ and $l$ but this volatility is necessary when the amount of data is so small. In particular, $r$ is completely undefined when $w = 0$ or $l = 0$, and in these cases we just take $r$ to be some arbitrary value $\sigma$. Consequently, all new players have $w = l = 0$ and hence have an initial rating of $\sigma$.

## 2 Fully rated players

Once a player has a full rating (from their $\omega^{th}$ game onwards), their rating is altered at the end of each game by $\Delta r$. $\Delta r$ is computed individually for each player, and is designed to be positive for the winner and negative for the loser; hence if a player enters the game with rating $r$, they finish with rating $r + \Delta r$. We define this change by

$$\Delta r = bcpl$$

with the following definitions.

$b$ is the base rating change, given by $b = \alpha + \beta|m|$, where $m$ is the margin of victory. Hence $\alpha$ controls the minimum gain for a win, and $\beta$ controls how many ratings points a player receives for each point they win the game by. Since we use $|m|$ (the absolute value of $m$, ignoring its sign), $b$ is the same for both players.

$c$ is the compensation factor for the original ratings difference. This is intended to reward players more for beating players rated above them, and punish them less for losing to such players. It is defined as

$$c = w - \frac{1}{1 + \exp(\gamma(r_{\text{him}} - r_{\text{me}}))}$$

where $r_{\text{me}}$ and $r_{\text{him}}$ are the ratings of this player and the other player respectively, and exp is the exponential function. $w$ is defined by

$$w = \begin{cases} 1 & \text{for the winner} \\ 1/2 & \text{for draws}^1 \\ 0 & \text{for the loser} \end{cases}$$

$c$ is the only factor that can be negative, and hence the sign of $c$ determines the sign of $\Delta r$. In particular, if the two players enter the game with the same rating, we simply take $c = w - 1/2$, which seems eminently sensible.

$p$ is the provisionality factor. Although the player for whom we are computing $\Delta r$ must be fully rated (see above), their opponent may not be. If the opponent is fully rated then simply set $p = 1$. Otherwise, this game is less statistically significant than we would like (since our estimate of the opponent's rating, and hence the value of $c$, is likely to be quite inaccurate.) Hence we must scale $\Delta r$ down. Assume the opponent has played $g$ games (and note that we must have $g < \omega$.) Then we define $p$ as:[2]

$$p = \frac{1}{\omega - g}$$

$l$ is is the length factor, which essentially measures how long the game was. Clearly a 15-round game is far more statistically significant than a 1-round conundrum attack, and the rating change reflects this. It is defined as

[2]It was a mistake in Apterous's implementation of this formula that caused the infamous "zero rating" bug which reset many people's ratings to zero!

$$l = \mu \ln(1 + n)$$

where $n$ is the number of rounds in the game. $l$ is also capped at $\lambda$, so that even enormously long games cannot cause huge ratings swings. The value of $\mu$ in Apterous is set so that $n = 15$ gives $l = 1$, so this factor essentially disappears for standard 15-round games; $\lambda$ is taken to be 2, so the combined effect is that all games of over 256 rounds have an equal $l$ value. In practice, such games are rare!

# 3 Apterous's constants

The particular value chosen for the constants obviously has a significant impact on the behaviour of the ratings sytem. The values used in Apterous are based on some particular desired behaviour, some guesswork, and some general experience of what works. They might well be changed in the future, but their current values are:

| Constant | Value |
|---|---|
| $\alpha$ | 20 |
| $\beta$ | 1/2 |
| $\gamma$ | 0.00575 |
| $\lambda$ | 2 |
| $\mu$ | $1/\ln 16 = 0.361\ldots$ |
| $\sigma$ | 600 |
| $\tau$ | 133 |
| $\omega$ | 11 |